Radii of Mirror Nuclei

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A nuclear model is used in which the nucleon density is a constant out to the core radius R and then decreases as $e^{-\alpha r}/r^2$, where $\alpha^2=8MW/\hbar^2$, in which W is taken as the average binding energy per nucleon. Values of R are determined from the measured energy differences between mirror nuclei by calculating the Coulomb energy difference on the above model. If one assumes that the density of the core is constant with changing A, then $R=1.36\times10^{-13}A^{\frac{1}{3}}/(1+3/\alpha R)^{\frac{1}{3}}$ and fits the data well. This also gives values of nuclear radii in agreement with those obtained from the recent electron scattering measurements of Lyman, Hanson, and Scott even for large A. For light elements, a good approximation to the average nuclear radius, $R+1/\alpha$, is $1.4\times10^{-13}A^{\frac{1}{3}}$. Nearly half of the nucleons are in the exponential tail.

AN important method for determining nuclear radii has been through the study of mirror nuclei.[1–4] A nuclear model is assumed, and then the Coulomb energy difference between the two mirror isobars is calculated. The only parameter is the radius and this is determined by fitting the experimentally measured energy difference. Apart from the model, the only assumption is that the n–n interaction is the same as the p–p interaction. Usually a constant density spherical nucleus is assumed, in which case the radii are given by $R=1.45\times10^{-13}A^{\frac{1}{3}}$. These radii are somewhat larger than those indicated by scattering measurements,[5–7] especially for heavy elements.

In this note it is assumed that the nucleus has a constant density core but that the nucleons penetrate to the outside field free region. The nucleonic wave function outside the core will then be identical to that characteristic of the deuteron, namely, $e^{-\frac{1}{2}\alpha r}/r$, where $\alpha^2=8MW/\hbar^2$ in which W is now taken as the average binding energy per nucleon. Thus in this model the density is constant out to a radius R and then falls off as $e^{-\alpha r}/r^2$.

The difference in Coulomb energy between the isobaric pair having atomic number Z and $Z+1$ for such a density distribution is given in terms of exponential integrals **Ei** by

$$\Delta E=\frac{6}{5}\frac{Ze^2}{R}\left[\frac{1+5(1+3/\alpha R)\,\mathbf{Ei}(-\alpha R)/e^{-\alpha R}-15\,\mathbf{Ei}(-2\alpha R)/\alpha R e^{-2\alpha R}}{(1+3/\alpha R)^2}\right].$$

The part outside the brackets will be recognized as the energy difference for a uniform spherical charge of radius R. The part inside the bracket which is plotted as F in Fig. 1 varies slowly with the parameter αR and is about 0.75 for typical values of αR.

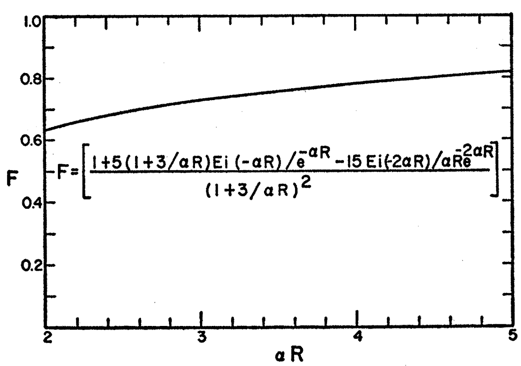

FIG. 1. The expression within the brackets of Eq. (1) of the text is plotted as a function of αR.

In Table I are listed measured energy differences,[8–13] as obtained from the positron spectra resulting from transition between mirror nuclei, and values of R resulting from calculations using the above expression. In Table II, values of b obtained from the relation $R=bA^{\frac{1}{3}}$ are given. It can be seen that b is surprisingly small, about 1.1×10^{-13} cm on the average, but not a constant. On the other hand, the quantity $R+1/\alpha$ is more nearly the average nuclear radius, and the values of $b'=(R+1/\alpha)/A^{\frac{1}{3}}$ given in the second column are nearly constant at 1.4×10^{-13} cm, the meson Compton wavelength. For comparison, values of $b''=r_0/A^{\frac{1}{3}}$ are also given where r_0 is the radius resulting from the uniform density model.

The ratio of the time a nucleon spends outside of R to the time it spends inside R is just $3/\alpha R$, and this ratio is roughly unity for light nuclei. Thus, since the

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TABLE I. Values of the core radius R as obtained from $R=6Ze^2F/5\epsilon$, where $\epsilon=E_{max}+2m_ec^2+(m_n-m_p)$ and F is given as a function of αR in Fig. 1. E_{max} is the end point in Mev of the positron spectra resulting from the transitions between mirror nuclei.

A	E_{max}(Mev)	$(1/\alpha)\times10^{13}$	$R\times10^{13}$	Ref.
11	0.97±0.01	0.86	2.15	11
13	1.24±0.01	0.83	2.46	12
15	1.68±0.01	0.82	2.55	13
19	2.20±0.08	0.81	2.98	10
23	2.82±0.09	0.80	3.21	10
	2.99±0.09		3.08	8
25	2.99±0.09	0.79	3.41	10
27	3.54±0.10	0.79	3.30	9
	3.48±0.10		3.35	8
29	3.63±0.07	0.78	3.56	3
31	3.85±0.07	0.78	3.71	3
33	4.13±0.07	0.78	3.80	3
	4.06±0.12		3.86	8
35	4.38±0.07	0.78	3.90	3
	4.43±0.13		3.86	8
37	4.57±0.13	0.78	4.00	8
39	5.13±0.15	0.78	3.86	8

nucleons spend about half of their time outside the nuclear core, we should expect considerable changes in the interpretation of some nuclear phenomena when considered on the basis of this model. The density in the core is $r_0^3/R^3(1+3/\alpha R)$ times that resulting from the uniform density model. In Fig. 2 is plotted the density distributed for carbon. It is reasonable to expect the core density to remain essentially constant as A varies. This implies that the core radius is given by the relation $R=1.36A^{1/3}/(1+3/\alpha R)^{1/3}$, where the constant 1.36 is the average of the values given in the third column of Table II. For Au, the above relation gives a value of the average radius, i.e., $R+1/\alpha$, which is consistent with the value determined by Lyman, Hanson, and Scott[7] from electron scattering measurements.

Jastrow and Roberts[14] have pointed out how a similar model can be used to explain the results of neutron and proton scattering experiments where, at low particle energies, the nucleon density tail is relatively opaque but, as the particle energy increases, the tail becomes more and more transparent. Thus the "size" of the nucleus depends in those experiments on the energy of incident particles. In the same way one must revise the interpretation of other experiments such as those related to the production and scattering of mesons in nuclei.

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TABLE II. Some values of the constant b as obtained from the relation $R=bA^{1/3}$ are given in column 1, where R is the core radius. The second column is for the average radius, $R+1/\alpha$. The third column gives the constant of the expression $R=b''A^{1/3}/(1+3/\alpha R)^{1/3}$ which would obtain if the core density remains constant with changing A. The last column gives $b_0=r_0/A^{1/3}$, where r_0 is the radius calculated on the basis of the simple uniform density model.

A	$b=R\times10^{13}/A^{1/3}$	$b'=(R+1/\alpha)\times10^{13}/A^{1/3}$	$b''=R\times10^{13}(1+3/\alpha R)^{1/3}/A^{1/3}$	$b_0=r_0\times10^{13}/A^{1/3}$
11	0.97	1.35	1.26	1.40
13	1.05	1.40	1.32	1.45
15	1.03	1.37	1.30	1.41
19	1.11	1.42	1.36	1.45
23	1.13	1.40	1.36	1.44
	1.08	1.36	1.32	1.39
25	1.16	1.43	1.39	1.47
27	1.10	1.36	1.32	1.40
	1.12	1.38	1.33	1.41
29	1.19	1.41	1.37	1.45
31	1.18	1.42	1.39	1.45
33	1.18	1.43	1.39	1.45
	1.20	1.44	1.41	1.47
35	1.19	1.43	1.39	1.45
	1.18	1.42	1.38	1.44
37	1.20	1.43	1.39	1.47
39	1.14	1.38	1.34	1.39

In conclusion, the arbitrariness of the model assumed must be emphasized. Its principal justification is the same as that for the constant density model, namely,

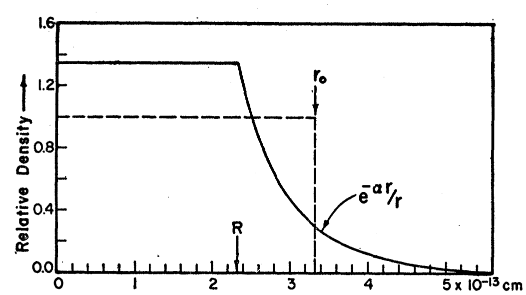

FIG. 2. The relative density of nucleons in carbon is plotted for the uniform density model (dashed curve) and for the core model with exponential tail (full curve).

its simplicity. A more sophisticated theory would give each nucleon its proper wave function, perhaps on the basis of the shell model.

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